

XXXVI. *Account of a new Instrument for measuring small Angles, called the prismatic Micrometer. By the Rev. Nevil Maskelyne, D. D. F. R. S. and Astronomer-Royal.*

Read Dec. 18, 1777. **P**RACTICAL astronomy was much benefited by the invention of the wire micrometer, for measuring differences of right ascension and declination: nor did it receive less advantage from Mr. SAVERY'S most ingenious invention of the divided object-glass micrometer, which has been rendered more commodious by the late Mr. JOHN DOLLOND'S application of it to the object-end of a reflecting telescope, or the present Mr. PETER DOLLOND'S application of it to the object-end of an achromatic refracting one.

But, valuable as the object-glass micrometer undoubtedly is, some difficulties have been found in the use of it, owing to the alterations in the focus of the eye, which are apt to cause it to give different measures of the same angle at different times. For instance, in measuring the Sun's diameter, the axes of the pencils of rays, which come through the two segments of the object-glass from

contrary limbs of the Sun, crossing one another at the focus of the telescope under an angle equal to that of the Sun's diameter, the union of the limbs of the two images of the Sun cannot appear perfect unless the eye be disposed to see objects distinctly which are placed at point of intersection. But if the eye be disposed to see objects distinctly, which are placed nearer the object-glass than the intersection is, the two limbs will appear separated by the interval of the axes of the pencils in that place; and if the eye be disposed to see objects distinctly, which are placed farther from the object-glass than the intersection is, the two limbs will appear to encroach upon each other by the distance of the axes of the pencils, after their crossing, taken at that place.

To explain this, let ov (plate XIX. fig. 1.) represent the centres of the two semi-circular glasses of the object-glass micrometer, separated to the distance ov from each other, subtending the angle oav , equal to the Sun's diameter, at the point a , which is the common focus of the two pencils of rays having oa and va for their axes, namely, those proceeding from contrary sides of the Sun, and passing through the contrary semi-circles; and let d be the eye-glass. It is evident, that if d be properly placed to give distinct vision of objects placed at the point a , the rays oa , va , as well as all the other rays belonging

to those pencils, will be collected into one point upon the retina of the eye; and consequently, the two opposite limbs of the two images of the Sun will seem to coincide, and the two images of the Sun to touch one another externally. But if the state of the eye should alter, the place of the eye-glass remaining the same, the eye will be no longer disposed to see the image formed at the point *a* distinctly, but to see an object placed at *ef*, nearer to or farther from the object-glass distinctly; and therefore an image will be formed on the retina exactly similar to the somewhat confused image formed by the rays on a plane perpendicular to their course at *ef*. Consequently, as the two cones of solar rays, *boa*, *cva*, formed by the two semi-circles, are separated or encroach upon one another at this point of the axis by the distance *ef*, the two images of the Sun will not seem to touch one another externally, but to separate or to encroach upon one another by the interval *ef*. The error hereby introduced into the measure of the Sun's diameter will be the angle *erf*, subtended by *ef* at *r* the middle point between *o* and *v*, which is to *ear* or *oav*, the Sun's apparent diameter, as *ae* to *er*, or even to *ar*, on account of the smallness of *ae* with respect to *ar*.

These considerations concerning the cause of a principal error that has been found in the object-glass micro-

meter led me to inquire, whether some method might not be found of producing two distinct representations of the Sun, or any other object, which should have the axes of the pencils of rays, by which they are formed, diverging from one and the same point, or nearly so: and it occurred to me, that this might be done by the refraction of a prism placed to receive part of the rays proceeding from the object, either before or after their refraction through the object-glass of a telescope. If the prism be placed without the object-glass, the rays that are refracted through it will make an angle with the rays that pass beside it equal to the refraction of the prism; and this angle will not be altered by the refraction of the object-glass afterwards. Consequently, two images of an object will be represented, and the prism so applied will enable us to measure the apparent diameter of any object, or any other angular distance which is equal to the refraction of the prism. But if the prism be placed within the object-glass, that is to say, between the object-glass and eye-glass, the angle measured by the instrument will vary according to the distance of the prism from the focus of the object-glass, bearing the same ratio to the refraction of the prism, as the distance of the prism from the focus bears to the focal length of the object-glass.

Let

Let ACB (fig. 2.) represent the object-glass, and d the eye-glass of a telescope, and PR a prism placed to intercept part of the rays coming from an object, suppose the Sun, before they fall on the object-glass. The rays EE proceeding from the Eastern limb of the Sun, and refracted through the object-glass ACB without passing through the prism, will form the corresponding point of the Sun's image at e ; and the rays ww proceeding in like manner from the Western limb of the Sun will be refracted to form the correspondent point of the Sun's image at w . But the rays $2E, 2E, 2W, 2W$, proceeding in like manner from the Eastern and Western limbs of the Sun, and falling on the prism PR , and thence refracted to the object-glass ACB , will, after refraction through it, form the correspondent points of the Sun's image at $2e, 2w$. Let the refraction of the prism be equal to the Sun's apparent diameter: in this case, at whatever distance the prism be placed beyond the object-glass, the two images of the Sun $w2e, 2w2e$, will touch one another externally at the point $e2w$; for the rays $2w, 2w$, proceeding from the Western limb of the Sun being inclined to the rays EE proceeding from the Eastern limb in the angle of the Sun's apparent diameter, will, after suffering a refraction in passing through the prism equal to the Sun's apparent diameter, emerge

from the prism and fall upon the object-glass parallel to the rays EE , and consequently will have their focus $2w$ coincident with the focus e of the rays EE , and therefore the two images of the Sun we , $2w2e$, will touch one another externally at the point $e2w$, and the instrument will measure the angle $EC2w$, and that only.

But if the prism be placed within the telescope, the angle measured by the instrument will be to the refraction of the prism as the distance of the prism from the focus of the object-glass is to the focal distance of the object-glass: or if two prisms be used to form the two images, with their refracting angles placed contrary ways, as represented in fig. 3. and 4. the angle measured will be to the sum of the refractions of the prisms as the distance of the prisms from the focus of the object-glass is to the focal distance of the object-glass. For let ACB (fig. 3.) represent the object-glass, and d the eye-glass of a telescope, and PR , RS , two prisms interposed between them, with their refracting angles turned contrary ways, and the common sections of their refracting planes touching one another at R . The rays proceeding from an object, suppose the Sun, will be disposed, by the refraction of the object-glass, to form an image of the Sun at the focus; but part of them falling on one prism, and part on the other, will be thereby refracted contrary ways, so as to form two equal images we , $2w2e$, which,
if

if the refractions of the prisms be of proper quantities, will touch one another externally at the point $e2w$. Let ECN be the axis of the pencil of rays EE proceeding from the Sun's Eastern limb; and wco the axis of the pencil of rays ww proceeding from the Sun's Western limb; and the point N the place where the image of the Sun's Eastern limb would be formed, and the point o where that of the Western limb would be formed, were not the rays diverted from their course by the refractions of the prisms. But by this means part of the rays EE , which were proceeding to N , falling on the prism PR , will be refracted to form an image of the Sun's Eastern limb at e , while others of the rays EE , which fall on the prism RS , will be refracted to form an image of the Sun's Eastern limb at $2e$. In like manner, part of the rays ww , which were proceeding to form an image of the Sun's Western limb at o , falling on the prism RS , will be refracted to form an image of the Sun's Western limb at $2w$ coincident with e , the point of the image correspondent to the Sun's Eastern limb; while others of the rays ww , which fall on the prism PR , will be refracted to form the image of the Sun's Western limb at w . The two images $w e$, $2w2e$, are supposed to touch one another externally at the point $e2w$. The ray EFR , which belongs to the axis ECN , and is refracted by the prism RR to e , undergoes the refraction $NR e$, which (because

cause small angles are proportional to their sines, and the sine of $NR e$ is equal to the sine of its supplement NRC) is to NCR as NC or ce is to NR or re . In like manner, the ray WGR , which belongs to the axis wco , and is refracted by the prism RS to zw or e , undergoes the refraction $OR e$, which is to $oc e$ as oc or ce is to RO or Re ; therefore, by composition, ORN the sum of the refractions $OR e$, $NR e$, is to ocn the sum of the angles $oc e$, $Nc e$, or the Sun's apparent diameter, as ce to re ; that is, as the focal distance of the object-glass to the distance of the prisms from the focus of the object-glass.

Or let the prisms PR , RS , be placed with their refracting angles P , S , turned from one another as in fig. 4.: the refraction of the prism PR will transfer the image of the Sun from ON to we , and the refraction of the prism RS will transfer the image ON to $zwze$, the two images $zwze$, we , touching one another externally at the point zew . Let ECN , wco , be the axes of the pencils of rays proceeding from the two extreme limbs of the Sun, and N , o , the points where the images of the Sun's Eastern and Western limbs would be formed by the object-glass, were it not for the refraction of the prisms; the ray EFR , which belongs to the axis ECN , and is refracted by the prism RS to ze , undergoes the refraction $NRze$; and the ray WGR , which belongs to the axis wco , and is refracted by

by

by the prism PR to w , undergoes the refraction ORw . Now $NCze$, part of the angle measured, is to $NRze$, the refraction of the prism RS , as Rw to cw ; and ocw , the other part of the angle measured, is to ORw , the refraction of the prism PR , in the same ratio of Rw to cw : therefore ocN , the whole angle measured, is to ORN , the sum of the refractions of the two prisms, as Rw to cw ; that is, as the distance of the prisms from the focus of the object-glass to the focal distance of the object-glass.

When the prisms are placed in the manner represented in fig. 3. the point e of the image we is illuminated only by the rays which fall on the object-glass between A and F , and the point zw only by the rays which fall on the object-glass between B and G . Now the angles CRF , CRG , equal to the refractions of the prisms, being constant, the spaces FC , CG , will increase in proportion as the distances RF , RG , increase, and the spaces AF , GB , diminish as much; and therefore, the images at the point of mutual contact ezw will be each illuminated by half the rays which fall on the object-glass when the prisms are placed close to the object-glass; but will be enlightened less and less the nearer the prisms are brought to the focus of the object-glass.

But

But when the prisms are placed in the manner shewn in fig. 4. the images at the point of contact, as the prisms are removed from the object-glass towards the eye-glass, will be enlightened with more than half the rays that fall on the object-glass, and will be most enlightened when the prisms are brought to the focus itself; for the point ze of the image $zwze$ will be enlightened by all the rays EE that fall on the object-glass between B and F , and the point w of the image wz will be enlightened by all the rays ww which fall on the object-glass between A and G . But the difference of the illuminations is not very considerable in achromatic telescopes, on account of the great aperture of the object-glass; as the greatest space FG is to the focal distance of the object-glass, as the sum of the sines of the refractions of the prisms is to the radius.

There is a third way, and perhaps the best, of placing the prisms, so as to touch one another along their sides which are at right angles to the common sections of their refracting planes. In this disposition of the prisms, the images will be equally enlightened, namely, each with half the rays which fall on the object-glass, wherever the prisms be placed between the object-glass and eye-glass.

From

From what has been shewn it appears, that this instrument, which may be properly called the prismatic micrometer, will measure any angle that does not exceed the sum of the refractions of the prisms, excepting only very small angles, which cannot be taken with it on account of the vanishing of the pencils of rays at the juncture of the two prisms near the focus of the object-glass; that it will afford a very large scale, namely, the whole focal length of the object-glass for the greatest angle measured by it; and that it will never be out of adjustment; as the point of the scale where the measurement begins (or the point of 0) answers to the focus of the object-glass, which is a fixed point for celestial objects, and a point very easily found for terrestrial objects. All that will be necessary to be done, in order to find the value of the scale of this micrometer, will be to measure accurately the distance of the prisms from the focus when the instrument is set to measure the apparent diameter of any object subtending a known angle at the centre of the object-glass, which may be easily found by experiment, as by measuring a base and the diameter of the object observed placed at the end of it, in the manner practiced with other micrometers: for the angle subtended by this object will be to the angle subtended by a celestial object, or very remote land object, when the

distance of the prisms from the principal focus is the same as it was found from the actual focus in the terrestrial experiment, as the principal focal distance of the object-glass is to the actual focal distance in the said experiment.

It will, I apprehend, be the best way in practice, instead of one prism to use two prisms, refracting contrary ways, and so divide the refraction between them (as represented in fig. 3. and 4.). Achromatic prisms, each composed of two prisms of flint and crown-glass, placed with their refracting angles contrary ways, will undoubtedly be necessary for measuring angles with great precision by this instrument: and I can add with pleasure, that I find by experiment made with this instrument, as it was executed by Mr. DOLLOND with achromatic prisms, ground with great care for this trial above a twelve-month ago, that the images after refraction through the prisms appear very distinct; and that observations of the apparent diameters of objects may be taken in the manner here proposed with ease and precision.

Two or more sets of prisms may be adapted to the same telescope, to be used each in their turn, for the more commodious measurement of different angles. Thus it may be very convenient to use one set of prisms for mea-

furing angles not exceeding 36', and consequently fit for measuring the diameters of the Sun and Moon, and the lucid parts and distances of the cusps in their eclipses; and another set of prisms to measure angles not much exceeding one minute, and consequently fit for measuring the diameters of all the other planets. This latter set of prisms will be the more convenient for measuring small angles, on account of a small imperfection attending the use of this micrometer, as before mentioned; namely, that angles cannot be measured with it when the prisms approach very near the focus of the object-glass, the pencils of rays being there lost at the point where the prisms touch one another.

Upon the principles that have been here explained, a prism placed within the telescope of an astronomical instrument, adjusted by a plumb-line or level, to receive all the rays that pass through the object-glass, may conveniently serve the purpose of a micrometer, and supersede the use both of the vernier scale and the external micrometer; and the instrument may then be always set to some even division before the observation. Thus the use of a telescopic level may be extended to measure with great accuracy the horizontal refractions, the depression of the horizon of the sea, and small altitudes and depressions of land objects. Time and experience

will doubtless suggest many other useful applications of this instrument.

A paper from the learned Abbé BOSCOVICH was read before this Society the ninth of last June, describing a similar contrivance as an invention of the Abbé ROCHON, in which the Abbé BOSCOVICH himself also claims some share; I therefore desire to acquaint this Society, that I communicated this invention to Mr. DOLLOND, and had it executed by him; and also shewed the instrument itself, so executed, to my esteemed friend ALEXANDER AUBERT, esq. fellow of this Society, a gentleman very well qualified to judge of things of this nature, above a twelve-month before the communication of the Abbé BOSCOVICH's paper, as will appear from their written attestations, drawn up at my desire, describing the particulars of the communication of this invention which I made to them so long ago. May I be permitted to remark, that this instrument having been executed by my directions, in several forms, by Mr. DOLLOND, between the months of March and August, 1776, and set up and tried at his house in the presence of several of his workmen, could not be considered as an absolute secret concealed from the public. However, I doubt not that the following attestations of Mr. AUBERT and Mr. DOLLOND will sufficiently prove my title to this invention of the prismatic micrometer;

meter; and I take this opportunity of exhibiting to the Society the instrument itself, mentioned in Mr. DOL-
LOND's letter as executed by himself according to my di-
rections, and sent to the Royal Observatory in the month
of August 1776.

Greenwich,
December 11, 1777.

TO THE REV. DR. MASKELYNE.

REV. SIR,

St. Paul's Church-yard,
Nov. 22, 1777.

ACCORDING to your desire I send the following particulars of the experiments which were made by your directions, for completing a new kind of micrometer for measuring small angles. About the beginning of April 1776, I received your first directions respecting this matter, which were to make two prismatic glasses or wedges of such angles that rays of light, which passed through them, should be refracted about $18'$ of a degree: these were to be placed between the object-glass and eye-glass of an achromatic telescope about 30 inches long. The angular edges of the two prismatic glasses were to be placed in contact with each other; they were to be moved in a parallel position from the object-glass to the
focus

focus of the eye-glass, and to be of such a size as to cover the aperture of the object-glass when brought close to it. By the refraction of these wedges two images were formed in the telescope, which were at the greatest distance (about 36') when the wedges were close to the object-glass, and approached as they were moved towards its focus, where they united; so that the whole focal distance of the object-glass was to be the length of the scale for measuring the angular distance of the two images formed in the telescope. When these wedges were applied, as above described, the two images were found to be coloured to a great degree, occasioned by the refraction of the wedges. This defect you directed me to remove by making the prismatic glasses or wedges achromatic, on the same principles as the achromatic object-glasses; and, after some difficulties, this was effected; the two images formed in the telescope appeared free from colours and distinct. The above experiments were made in a rough wooden tube, with an inconvenient method of moving the wedges by hand: in this state it was when shewn to ALEXANDER AUBERT, esq. F. R. S. towards the end of May, 1776; after which you desired to have it done in a more compleat manner, in a brass tube, with a means of turning the tube round to take angles in different directions, and a method of moving

moving the wedges with a screw. This was completed about the middle of August in the same year, and then sent to the Royal Observatory. I have the honour to be,

REVEREND SIR,

Your obedient humble servant,

PETER DOLLOND.

I HEREBY certify, that in the month of May, 1776, the Rev. Mr. MASKELYNE, Astronomer-royal, produced to me, at Mr. DOLLOND'S house in St. Paul's church-yard, and in his presence, as a new invention of his own, an instrument for measuring small angles, consisting of two achromatic prisms or wedges applied between the object-glass and eye-glass of an achromatic telescope about 30 inches long, by moving of which wedges nearer to, or farther from, the object-glass, the two images of an object produced by them appeared to approach to, or recede from, each other, so that the focal length of the object-glass became a scale for measuring the angular distance of the two images.

London,
Nov. 27, 1777.

ALEX^R. AUBERT.



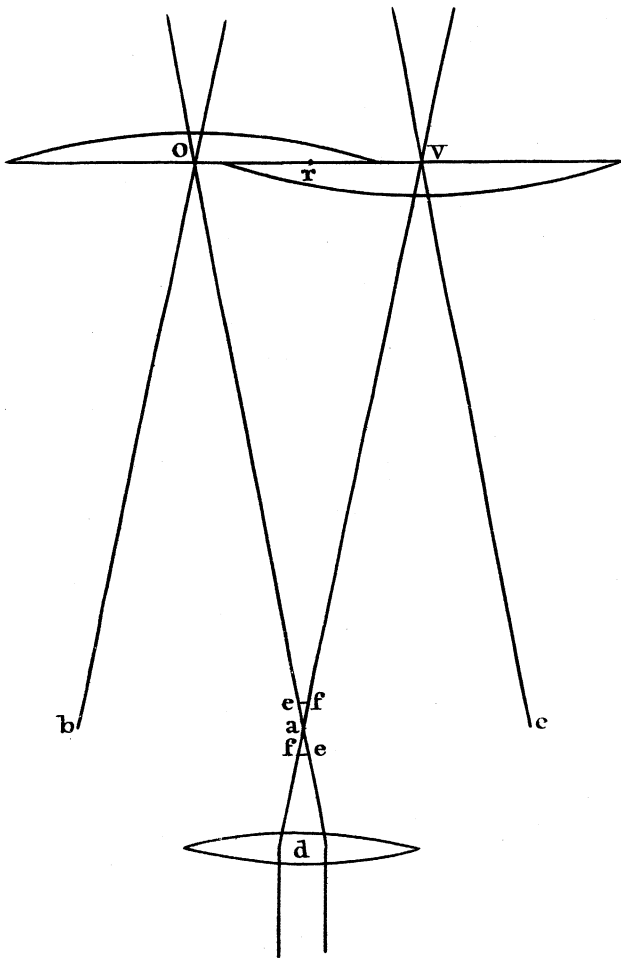


Fig. I.

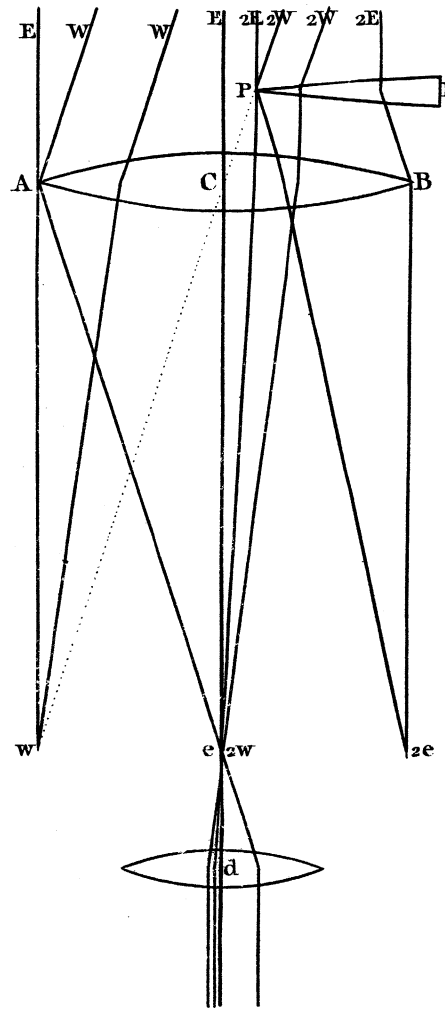


Fig. II.

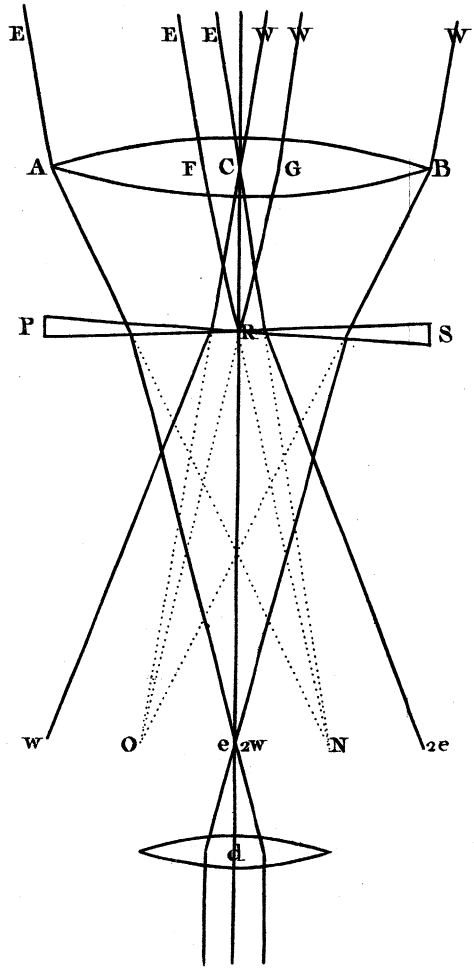
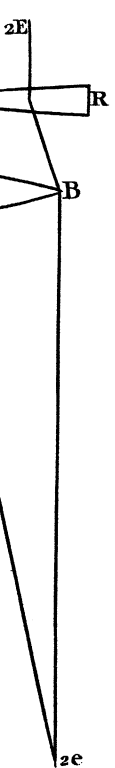


Fig. III.

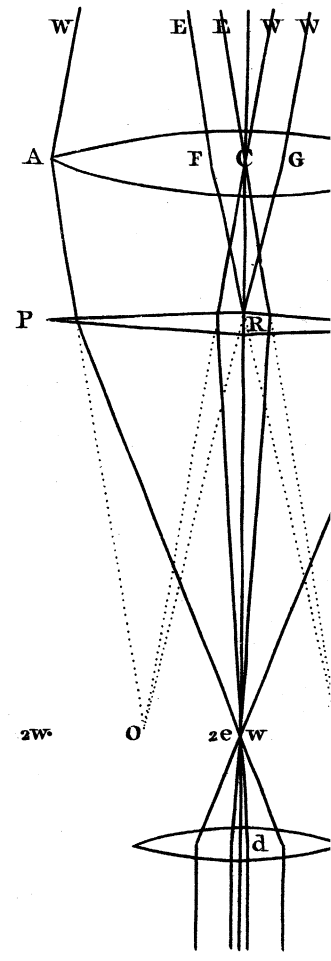
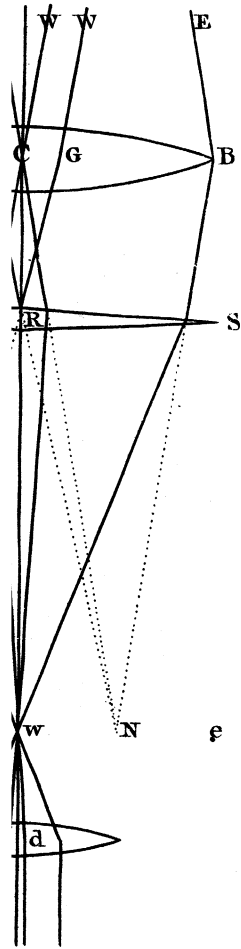


Fig. IV.



IV.

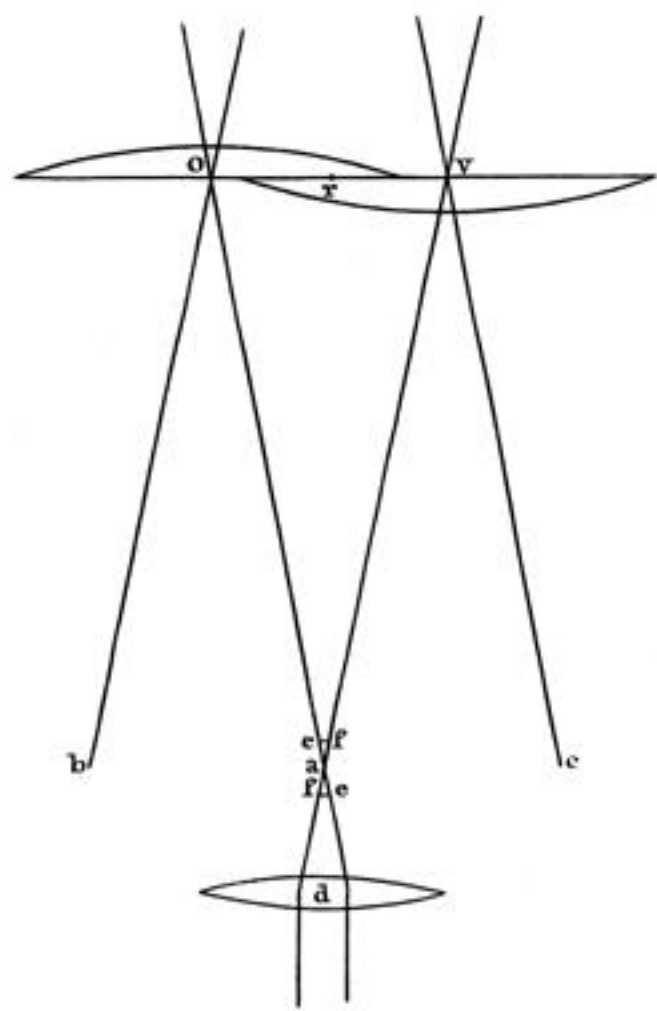


Fig. I.

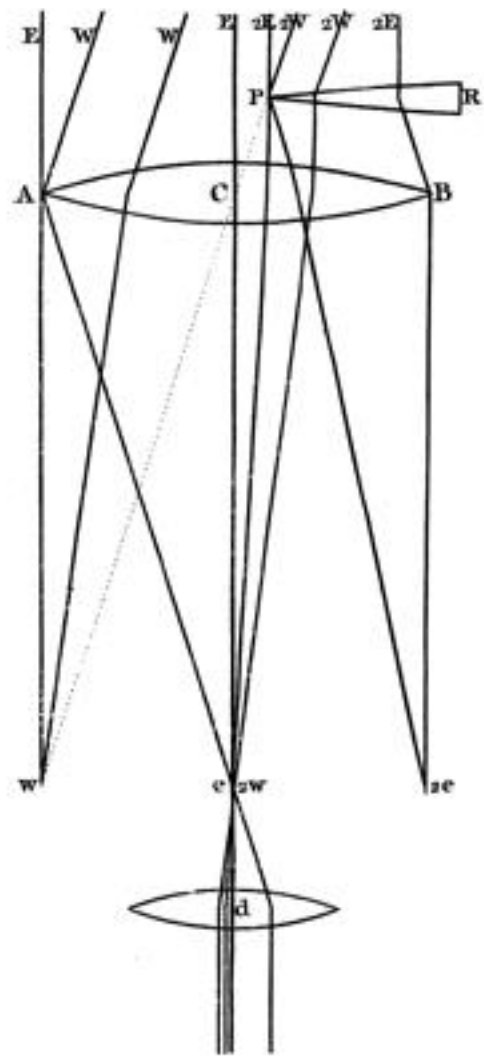


Fig. II.

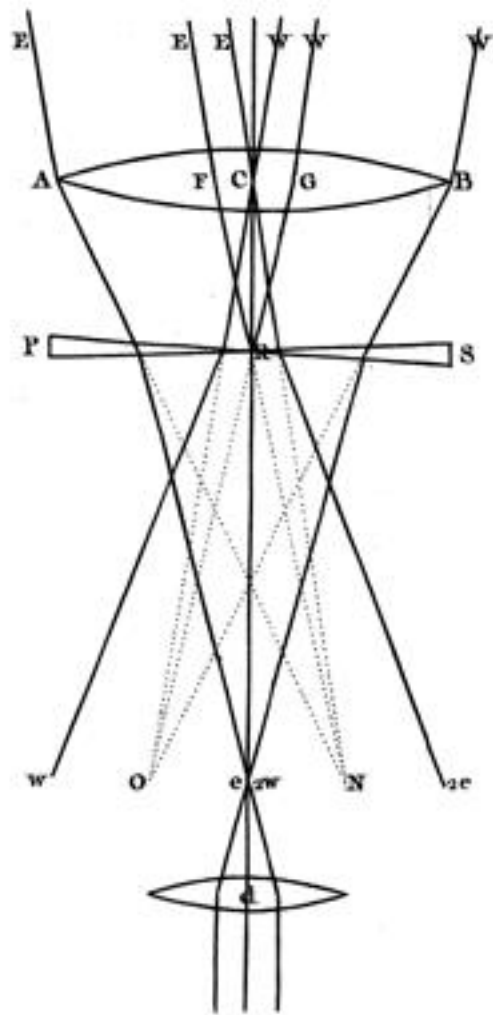


Fig. III.

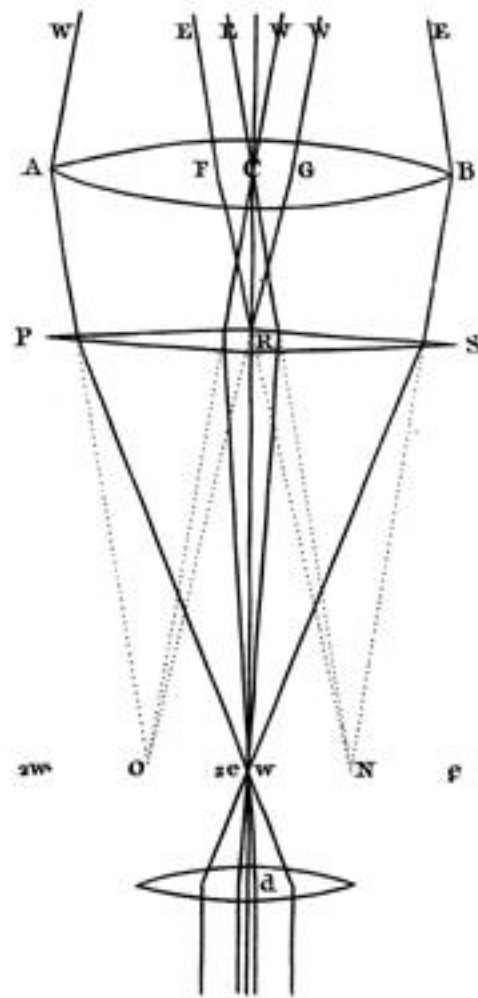


Fig. IV.